Twin Worlds

First Contact - The Long-Held Dream

Richard Arthur

The Complete Appendices

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Full set of Appendices

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Appendix 1
The Geos-Duplos Solar System

The Sun is very much like Earth's. It is a type G2V star of diameter 1,408,415 km and a mass of 2.05965 x 10^{30} kg which is 34,500 Earths. Its mean angular size from Geos-Duplos is 32.22 arc-minutes. The binary planets Geos and Duplos orbit at a mean distance of 156.261 million kilo-metres and are the only planets in the habitable zone. There are four other planets in the solar sys-tem, one rocky inner planet, Thermos, two gas giants, Zuvos and Sépanos, and one outer ice planet, Ultimos. There are also several moons. Fig A1.1 illustrates the planets and the three largest moons of Zuvos, indicating their relative sizes. Sépanos also has a large moon, Atlas, which is not represented in Fig A1.1 but is just slightly smaller than Proxis. The tables below list various proper-ties of the planets and main moons, using Earth measurements[1].

Table A1.T1 Planet Properties

Planet	Thermos	Duplos	Geos	Zuvos	Sépanos	Ultimos
Mass (kg)	1.54×10^{24}	3.36×10^{24}	8.71×10^{24}	1.31×10^{27}	3.15×10^{26}	3.06×10^{22}
Mass (Earths)	0.258	0.563	1.459	219.357	52.764	0.00513
Diameter (km)	8132	10403	14550	126576	91708	3150
Density (gm/cm^3)	5.47	5.7	5.4	1.26	0.78	1.87
Surface Gravity (g)	0.634	0.845	1.119	2.255	1.018	0.084
Escape Velocity (km/s)	7.11	9.28	12.64	52.73	30.27	1.61
flatness	0.00662	0.0128	0.00357	0.0257	0.1094	0.00254
Rotatation Period (h/d)	52.618 d	3 d	2 d	10.02 h	12.31 h	3.92 d
Rotation Speed (km/h)	20.28	453.92	952.30	40152	24306	104.39
Orbital Characteristics						
Semi-Major Axis (AU)	0.5134	1.0044	1.0044	5.361	11.5309	33.6636
Semi Minor Axis (AU)	0.5103	1.0044	1.0044	5.3567	11.508	33.017
Aphelion (AU)	0.5698	1.012	1.012	5.5755	12.2573	40.2279
Perihelion (AU)	0.5072	1.0044	1.0044	5.3525	11.4851	32.3825
Eccentricity	0.11	0.008	0.008	0.04	0.063	0.195
Period (d/y)	131.545 d	359.999 d	359.999 d	12.352 y	38.40 y	191.57 y
Mean orbital velocity (km/s)	42.33	30.35	30.35	13.11	8.93	5.185

1 1 Astronomical Unit (AU) is the mean radius of the Earths orbit, 149.598 million km. g = 9.81 m/s^2 is Earth's Sur-face Gravity and a mass of 1 Earth is 5.97×10^{24} kg. Rotations are measured in (Earth) hours or days, and orbit peri-ods in days or years.

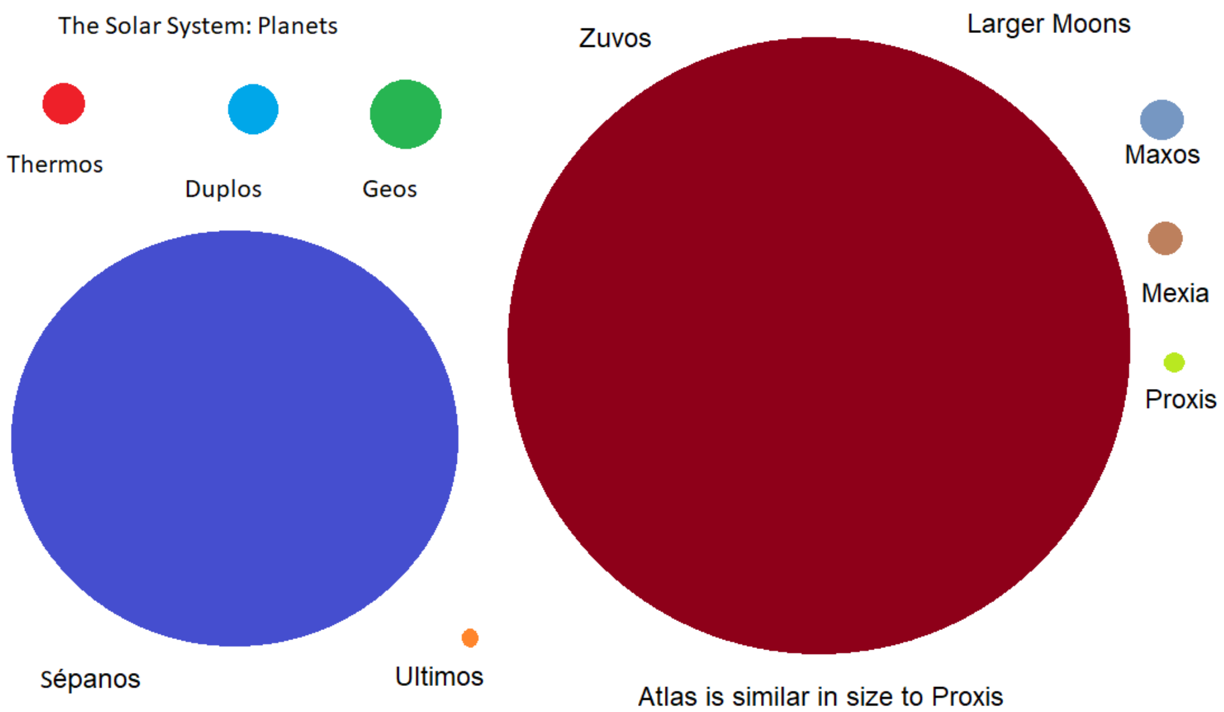

Fig A1.1 The Solar System

Table A1.T2 gives similar Data on the larger moons of Zuvos and Sépanos, and the moons of Maxos.

Table A2.T2 Moons

Moons of			**Zuvos**			**Sépanos**	**Maxos**	
Name	**Proxis**	**Archos**	**Mexia**	**Maxos**	**Talos**	**Atlas**	**Maxos.1**	**Maxos.2**
Mass (kg)	1.63×10^{23}	3.53×10^{19}	7.62×10^{23}	1.28×10^{24}	8.68×10^{21}	9.57×10^{22}	7.62×10^{17}	5.26×10^{16}
Diameter (km)	4500	250	7200	9500	2100	4300	65	27
Density (gm/cm^3)	3.42	4.31	3.9	2.85	1.79	2.3	5.3	5.1
Gravity (g)	0.219	0.015	0.4	0.386	0.054	0.141	0.005	0.002
Escape Velocity	3.11	0.194	5.314	5.994	1.05	2.437	0.056	0.023
flattening	0.00798	0.0143	0.0124	0.00609	0.00805	0.00557	N/A	N/A
Rotation			Synchronous				Unknown prob. synchronous	
			Orbital Characteristics					
Semi-Major apsis (km)	404000	638000	1018000	1635000	2160000	1172000	15000	32000
Period (d)	1.99	3.95	7.96	16.2	24.6	20.05	0.46	1.42
Eccentricity	0.038	0.005	0.041	0.0003	0.09	0.12	0.007	0.0001

We now come to the Geos-Duplos system, As well as the two planets in orbit around their barycentre, there are two moons. Selenos is the larger and orbits in a near-circular orbit some way out. Eclos is a captured asteroid and travels in an eccentric orbit which is highly inclined to the plane of the Geos-Duplos orbits.

The Geos-Duplos orbit is itself inclined at 2° to the ecliptic, here defined as the plane of the orbit of the Geos-Duplos barycentre about the sun. The two planets orbit about the barycentre in a 'dumb-bell' orbit in a similar manner to Pluto and Charon, as distinct from the intersecting ellipses common with binary stars. Although there is tidal locking between the planets, it is not in 1:1 ratio like the Moon with its orbit of Earth, or the Jovian and Zuvian moons. But in a simple ratio, as are Mercury and Thermos. The Geos-Duplos year is almost exactly three hundred and sixty Earth days long. The period of their rotation about their barycentre is twelve Earth days, during which time Geos rotates six times and Duplos four times, giving local days, of two and three Earth days. Shadowing by the other planet is such that the light and darkness pattern is complicated and both Geosians and Duplosians are very flexible about their rest times.

Table A1.T3 gives various parameters of the planets and moons and their orbits.

Table A1.T3 The Geos -Duplos System

	Geos	**Duplos**	**Selenos**	**Eclos**	**Space Station**
Mass (kg)	8.71×10^{24}	3.36×10^{24}	2.09×10^{22}	9.19×10^{18}	N/A
Diameter (km)	14550	10403	2217	154	N/A
Density (gm/cm^3)	5.4	5.7	3.67	4.8	N/A
Gravity (g)	1.119	0.845	0.116	0.011	N/A
Escape Velocity (km/s)	12.637	9.283	1.587	0.126	N/A
flattening	0.00357	0.0128	0.024	N/A	N/A
Axial tilt to solar orbit	17.61°	17.61°	N/A	N/A	N/A
Rotation (d)	2	3	Synchronous	Unknown	N/A
Orbital Characteristics (all distances in km, times in days)					
Semi-Major Apsis	97095	251669	1072000	2531000	325589
Semi-Minor Apsis	97095	251668	1071999	1373286	325588
Closest to Station from Surface	73920	73920	N/A	N/A	N/A
min separation from Geos	N/A	348761	974905	N/A	N/A
max separation from Geos	N/A	349496	1169095	N/A	N/A
Period of orbit	12	12	89.61	325.11	15
Eccentricity	0.0021	0.0021	0.0013	0.84	0.0021
Inclination of G-D orbit to Ecliptic	15.5°	15.5°			
Inclination to G-D orbit	0	0	0.23°	43.17°	0

One aspect that comes up in Arté's calculations is that of the angular size of various bodies in the solar system as seen from Duplos. Table A1.T4 below gives the angular size in Arc-minutes of various bodies as seen from Geos-Duplos.

Table A1.T4 Angular Size from Geos-Duplos (all in arc-min)

Angular Size	Sun	Thermos	Zuvos	Selenos	Eclos
Mean	32.22	4.97	0.16	7.18	N/A
Max	N/A	15.23	0.663	9.37	1.31
Min	N/A	N/A	N/A	5.82	0.11
with x40 scope			26.52		

For comparison the angular size of both the Sun and Moon from Earth is around 30 arc-min. Of course Geos and Duplos each appear much larger in each other's skies, at about 143 arc-min (Geos from Duplos) and 102 arc-min (Duplos from Geos).

Much of Arté and Rastu's energies were spent on teasing out orbits. Fig A1.2 is a two dimensional schematic of the orbits in the system. Fig A2.3 shows the apparent orbits of Geos (circular) and Selenos (loopy) as seen from Duplos over one full 89.61 day Selenos cycle. The Geos orbit is the inner circle which as already stated completes every twelve days.

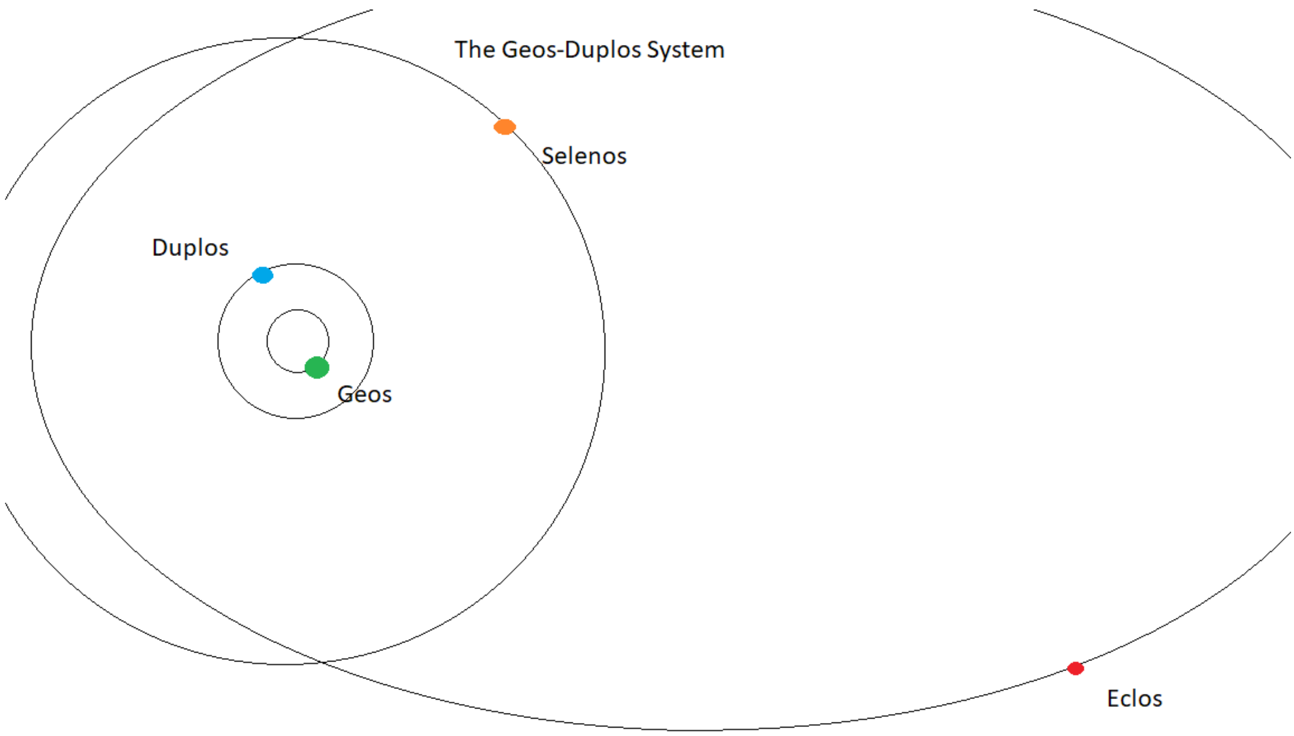

Fig A1.2 The Geos-Duplos System.

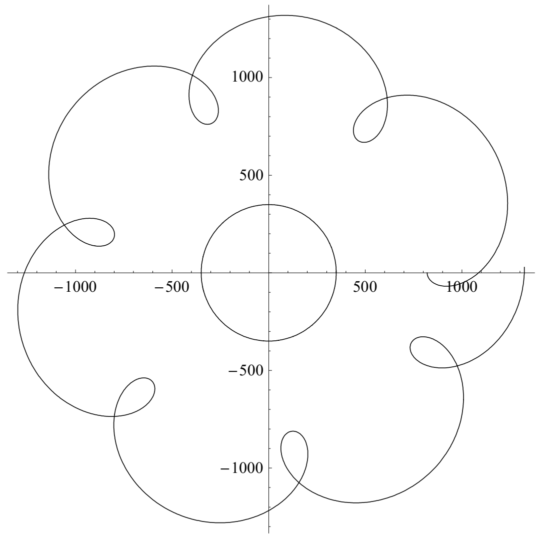

Fig A1.3 The apparent orbits of Geos and Selenos as seen from Duplos

(scale in Megametres)

Appendix 2
Duodecimal numbers

On Earth we count and express our numbers using the decimal place value number system, that is we work base ten. In continental Duplos they use the same system so there is no difficulty there. On Isla, the isolated island culture on Duplos, which is more primitive than the continent, they use a system more akin to Roman Numerals. Appendix 6 covers this in more detail.

On Geos they have adopted a duodecimal system. This makes a lot of sense in many ways, but can be confusing for us. Rather than introduce twelve completely new symbols, I have decided to use the ten we are familiar with and two additional ones, A and B to denote ten and eleven respectively. In order to limit confusion I shall put a number in ***bold italics*** whenever it is to be interpreted as a duodecimal number. Thus while 10 will mean ten, ***A*** will also mean ten and ***10*** will mean twelve.

Powers of twelve are the numbers consisting of a one followed by a number of zeros. Thus ***100*** is 144, ***1000*** is 1728 etcetera. The number ***1728*** is the duodecimal for 2768. ($1 \times 12^3 + 7 \times 12^2 + 2 \times 12 + 8$). For fractions, the familar form of numerator/denominator is the same as we are used to, it's just that the numbers have to be iinterpreted as duodecimals. However when we use the analagous notation to our decimals, we have to remember than now it is not a decimal point but a duodecimal point and the duodigits after the point represent twelfths, one hundred and forty-fourths and so on. Thus ***3/4*** is the same number as 3/4 but if we want to put it in the 'decimal' notatation it won't be ***0.75*** but ***0.9. (***9 twelfths is the same as three quarters, ***0.75*** would be 7/12+ 5/144 = 0.618055555..). Just as with decimals all rational numbers (fractions) in the duodecimal form will either stop after a finite number of places or eventually the last part will be recurring. The fractions that give finite duodecimals will be those which in their lowest terms have only factors of powers of 2 and 3 in the denominator (just as for decimals the denominator has to only have factors of powers of 2 and 5).

One of the common usages in the main book is for dates on Geos. The events that are being described occur around the year ***1600*** in the Geos calendar, which is close to 2600 decimal. Duplos doesn't have a calendar for us to number its years, and in part three of the book as the events on both planets come together, I have expressed dates in the decimal equivalent of the Geos dates. Thus the final chapter occurs in the year ***161A*** or 2614.

Finally on language. The numbers one to twelve are as in English. However after that a different set of number words is required. Instead of the teens, we have the tweens; onetween, twoteen, thirtween to ninetween, tentween, leventween. Then we go through the twentwy's to leventwy's until we reach a twundred for ***100*** or 144. Finally a twousand for ***1,000*** $=12^3 = 1728$ and a twillion for ***1,000,000*** $= 12^6$.

On Earth we use micro, milli, kilo, mega as prefixes to denote powers of 10^{-3} and 10^3. On Geos they have a similar system for powers of 12^{-3} and 12^3. Thus swega, swili, twilo and twega means respectively multiply by 12^{-6}, 12^{-3}, 12^3 and 12^6. Finally we have pertwent and pertwentage for the base twelve version of percent and percentage.

Appendix 3
Weights and Measures on Geos and Duplos

People on Geos and Duplos

Just as on Earth in ancient times our systems of weight and measures owed some reference to measurements on people, the same is true on each of these planets, so by way of introduction. I will cover the basic dimensions of Geosians and Duplosians.

On Geos there is no gender dimorphism, apart from the obvious. Both males and females have an average height of 1.53 metres (a tiny bit over 5 ft) and an average mass of 75 kg (165 lb). That is, on Earth, an average Geosian would tip the scales at 75 kg, and a 75 kg Earthling would weigh the same as the average Geosian on Geos (nearly 84 kg weight).

Geosians have a thickset stocky build compared to us on Earth, and female breasts are small. Fashions of both clothing and hairstyle are unisex. So it is impossible to tell male from female when dressed. Variation in height, girth and natural weight[2] are lower than on Earth. Height is approximately normally distributed whith standard deviation of just 3.5 cm (1.38"). Thus 68.3% of Geosians are between 1.495 and 1.565 metres tall and 95.44% are between 1.46 and 1.6 metres tall.

On the other hand Duplosians are tall and thin and do show gender dimorphism. Their limbs are also proportionately longer than ours. Duplosian men have an average height of 2.2 metres (7' 2.6") and average mass of 72 kg (159 lb). Duplosian women have an average height of 2.3 metres (7' 6.55") and an average mass of 73 kg (161 lb). The standard deviation of height for both men and women is 10 cm (4").

Measures in general

On Geos, as on Earth, the traditional definitions of measurement have been replaced by references to physical constants in a way that preserves the old nomenclature while ensuring the exact quantities are precisely defined. Duplos is less advanced. On the continent they have exact standards based on standard physical objects. On Isla things are a bit more rough and ready, but there are standard rulers and weights kept in the more important temples. Those in the principal temple at Rivermouth are increasingly seen as the base standard.

Time

The **year** and **period** are common to both planets. The period is the time that Geos and Duplos complete their orbits around each other at exactly 12 Earth days, while the year is 359.999 Earth days, almost exactly 30 periods long. Most other time units are divisions or multiples of the period, influenced by binary, decimal or duodecimal divisors. The only exception is the **selenth**, based on the orbit of Selenos, at 89.614 Earth days. On Geos and continental Duplos, the selenth has been decoupled from the motion of Selenos, and refers to a quarter of the year. (90 days). On the island of Isla, they stick to the lunar calendar but adjust at New Year to maintain reasonable alignment between the seasons of the year and selenths. Table A3.T1 below shows the time units and comparisons with Earth.

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2 Just as on Earth some Geosians (and indeed Duplosians) are fatter or thinner, lighter or heavier due to lifestyle choices.

Table A3.T1 Time Units

	Geos	Duplos Continent	Duplos Isla	Earth
Geos: Tick	1 tick	1.37 dusec	2.808 isec	2.778 sec
Tock	72 ticks ***(60)***	2.47 dumin	3.16 imin	3 min 20 sec
Meal	72 tocks ***(60)***	4.44 hora	3.555 ihor	4 hours
Watch	12 meals ***(12)***	53.33 hora	42.67 ihor	2 days
Period	6 watches ***(6)***	1 period	1 period	12 days
Work	5 periods ***(5)***	5 periods	5 periods	60 days (2 mnth)
Selenth	1.5 works ***(1.6)***	7.5 periods	(89.614 days)	90 days
Year	30 periods ***(26)***	30 periods	30 periods	360 days
Life	72 years ***(60)***	72 years	72 years	70.97 years
Mem	144 years ***(100)***	144 years	144 years	141.94 years
Duplos (Dusec)	0.729 ticks ***(0.88B9)***	1 dusec	2.047 isec	2.025 sec
Dumin	29.16 ticks **(25.1B)**	40 dusecs	1.28 imin	1 min 21 sec
Horum	16.2 tocks ***(14.2497)***	40 dumins	0.8 ihor	54 mins (0.9 h)
Phase	1.5 watches ***(1.6)***	80 hora	1 phase	3 days
Period	1 period ***(1)***	4 phases	4 phases	12 days
Selenth	1.5 works ***(1.6)***	7.5 periods	(89.614 days)	90 days
Year	30 periods ***(26)***	4 Selenths	30 periods	360 days
Century	100 years	100 years	100 years	98.57 years
Isla (Isec)	0.356 ticks	0.488 dusec	1 isec	0.989 sec
Imin	22.78 ticks	0.78125 dumin	64 isecs	1.055 mins
Ihor	20.25 tocks	1.25 hora	64 imins	1.125 hours
Phase	1.5 watches ***(1.6)***	1 phase	64 ihor	3 days
Period	1 period ***(1)***	4 phases	4 phases	12 days
Selenth	0.99 selenths ***(0.BA8)***	0.99 selenths	1 selenth	89.164 days
Year	30 periods ***(26)***	4 Selenths	30 periods	360 days
Earth : Second	0.36 ticks	0.494 dusecs	1.011 isecs	1 sec
Minute	21.6 ticks	29.63 dusecs	56.89 isecs	60 secs
Hour	18 tocks	1.11 hora	0.89 ihor	60 mins
Day	0.5 watches	0.33 phases	0.33 phases	24 hours
Month (1/12 year)	2.536 periods	2.536 periods	2.536 periods	30.435 days
Year	1.0145 years	1.0145 years	1.0145 years	365.22 days

Obviously the terms year, selenth and period come from astronomical phenomena, as do phase and watch which are the periods of rotation of the two planets. The others are clearly derived from them. On Geos, the terms meal and work derive from a meal being a standard interval between mealtimes and a work being the usual time period for payment of salaries, although casual labourers are often paid per period. Because of possible confusion, in the books I put the words **meal, watch and work** in bold typeface when used as a time unit.

Length and Distance

The base unit of length and distance in all three cultures was set by reference to people, in the way that such units as foot, yard and mile were established on Earth. On Geos the base standard is the 'man' which was defined by the average height of a Geosian. 1 **man** = 1.53 metres = 5' 0.236". All the other length and distance units derive from this via multiples of 12.

On Duplos the base unit is the 'stride', like the Earth yard. The value of a stride on the continent is slightly longer than on Isla. Where I need to distinguish I shall write the Isla version as 'istride'. A **stride** is 1.25 metres = 4' 1.21", an **istride** is 4' = 1.2192 metres. All other length and distance units are derived from these by simple multiples or factors. On the continent these tend to be mixed binary and decimal. On Isla, binary is the norm. Table A3.T2, compares the measurments with those of Earth

Speeds and Areas

These are all derived from the units of length and time but some quantities are worth noting.

Speed

Geos:

speed is measured in twilomen per **meal** (tpm) or races per tock (rpt). In most towns the speed limit is ***60*** tpm which works out as 118.31 miles in 4 hours or 29.58 mph. Converting from tpm to rpt is simple, just divide by six - very easy in base twelve. (As seen in chapter 10, twilomen per tick (tpt) is also used for some astronomical velocities).

Continental Duplos uses 'walk' as a measure of speed as well as distance. Walk's actual derivation was from the typical distance covered in a quarter of a horum. As a speed walk means four walks per horum (= 11.11 kph = 6.94 mph). Similarly Duplosians use the terms Jog and Run for eight and sixteen walks per horum respectively.

Actually the Run which equates to running a mile in 2.16 minutes is optimistic for most Duplosians. Their historic community record (they don't have world competitions) for the half walk (1.25 km) is 1 dumin 8 dusec which equates to a speed of 1.042 Runs. For the longer distance of one walk the accepted record is 2 dumin 21 dusec, which is just a bit too slow.

Area:

Geos:

A 'space' = a square side - a sort of standard building plot, a 'field' = a square race (quite a large field, in our units it works out at about 57600 square yards or 11.9 acres or 4.76 hectares). This is the main context that 'side' and 'race' get used. Otherwise, 'race' is so-called because it is a historic distance for a running race on Geos. For larger areas a square twiloman is about 2.7 square miles.

Duplos:

Continent: the only specific unit of area is the 'plot' which is a square block (625 sq metres or 1/16 hectares or 0.15625 acres). Large areas are expressed in square walks - 625 hectares or 2.41 square miles.

Isla: there are two area units, a 'square' is a square foot and an 'ifield' is a square furrow - approximately 6 acres or 2.4 hectares.

Isla doesn't have any specific terms for speed.

Table A3.T2 Length & Distance

Geos	Duplos Continent	Duplos Isla	Earth metric	Earth imperial
1 Slivver	No equivalent	No equivalent	74 micrometres	2.9 Thou
144 ***(100)*** slivvers = 1 Chink	0.34 div	2.231 smidges	1.0625 cm	0.404"
12 ***(10)*** chinks = 1Chunk	4.08 div	3.346 thumbs	12.75 cm	4.85"
12 ***(10)*** chunks = 1 **Man**	1.224 strides	1.255 istrides	1.53 m	5' 0.24"
12 ***(10)*** men =1 Side	14.688 strides	15.05 istrides	18.36 m	20^{x} 0' 0.236"
12 ***(10)*** sides = 1 Race	176.256 strides	181.71 istrides	220.32 m	240^{x} 2' 10"
12 ***(10)*** races = 1728 ***(1000)*** men = 1 Twiloman	1.0575 walks	2.169 imiles	2.64384 km	1.643 miles
2.94 chinks	1 Division	0.82 thumbs	3.125 cm	1.23"
2.45 chunks	10 div= 1 Length	1.025 ft	31.25 cm	1' 0.3"
0.81 men	4 lengths =1 **Stride**	1.025 istrides	1.25 m	4' 1.21"
1.634 men	2 strides = 1 Pace	2.05 istrides	2.5 m	8' 2.43"
16.34 men	10 paces = 1 Block	20.51 istrides	25 m	27^{x} 1' 0.25"
0.9456 twilomen	1000 paces = 1 Walk	2.002 imiles	2.5 km	1.554 miles
64.55 slivvers	0.1524 div	1 Smidge	4.7625 mm	.1875"
3.586 chinks	1.2192 div	8 smidges = 1 thumb	38.1 cm	1.5"
2.39 chunks	0.97536 lengths	8 thumbs = 1 foot	30.48 cm	1'
0.7969 men	0.97536 strides	4 ft = 1 i**Stride**	1.2192 m	4'
102 men	124.85 strides	128 istrides = 1 Furrow	156.06 m	170^{x} 2'
0.4722 twilomen	0.4994 walks	1024 istrides = 8 furrows = 1 imile	1.249 km	0.776 miles
7.84 chunks	0.8 strides	3.28 ft	1 metre	1^{x} 0' 3.37"
0.37824 twilomen	0.4 walks	0.801 imiles	1 km	0.6215 miles
0.6085 twilomen	0.6436 walks	1.289 imiles	1.609 km	1 mile

Volume

All these societies have adopted some volumetric measures from historic liquid measures otherwise they will use cubic, as we do, in the same way as square for areas. Table A3.T3 below summarises the volumetric measures.

Table A3.T3: Volumetric measures

Geos	Duplos Continent	Duplos Isla	Earth metric	Earth imperial
1 Swilijug = 12^{-3} jugs	5×10^{-3} scoops	2×10^{-3} ipints	1.2 mls	0.073 cu in 0.0253 pints
1 Cup	0.7242 scoops	0.2981 ipints	172.72 mls	0.3039 pints
1 **Jug** = 12 ***(10)*** cups = 1 cu chunk	1.0863 pots	3.5775 ipints	2.072 litres	3.646 pints
1.414 cups	1 Scoop	0.4117 ipints	238 mls	0.42 pints
5.656 cups	1 doop = 4 scoops	1.6468 ipints	954 mls	1.6783 pints
0.9205 jugs	1 Pot = 2 doops	3.2935 ipints	1.90735 litres	3.3566 pints
1.8411 jugs	1 Dugal = 2 pots	6.587 ipints	3.8147 litres	6.713 pints
14.729 jugs	1 **Dukin** = 8 dugals = 1 cu length	6.587 igals = 1.0777 cu ft	30.518 litres	6.713 gallons
117.83 jugs	1 dubarr = 8 dukins	1.6466 ibarrels	244.14 litres	53.7 gallons
3.3543 cups	2.4292 scoops	1 **ipint**[3]	579.17 ml	1.0192 pints
2.2361 jugs	1.2146 dugals	1 igallon = 8 ipints	4.633 litres	1.0192 gal
71.56 jugs	4.858 dukins	1 ibarrel = 32 igal	148.27 litres	32.6144 gal
13.668 jugs	0.928 dukins	1 cubic foot	28.32 litres	1 cu ft = 6.23 gal
3.37 cups	2.383 scoops	0.9812 ipints	568.26 ml	1 pint
5.929 cups	1.0486 doops	1.7266 ipints	1 litre	1.7598 pints
2.2467 jugs	1.1917 dugals	0.98192 igal	4.546 litres	1 gallon

Mass & Weight

These two quantities are linked on Geos just as on Earth. Of course on Geos their local g defines the relationship. Surface gravity on Geos is 1.119 times that on Earth or 10.98 m/s^2. This translates as 55.38 men/tick^2 which in the Geosian duodecimal system is ***47.47*** *men/tick^2.*

As with linear measurement the measurement of weight and thence mass is based on a 'standard' person at 75 kg. Confusingly, Geosians use the same term 'man' as for linear measurement, relying on context to distinguish. For smaller and larger quantities other terms are used for some duodecimal multiples/factors of the base man.

For astronomers, **1** Geos = 1.4588 Earths, 1 Earth = 0.685 Geos.

Continental Duplosians have not yet worked out the physics of weight and mass, their science is pre-Newtonian. However they do have the concept of weight under Duplosian gravity[4]. On the continent their standard weight is that of of a doop of water (1 dupound - dp), which under Earth gravity would be 976.5625 grams or 2.152 lb. (On duplos this weighs 1.82 lb or 0.825 kg). The average mass of Duplosians is 73 kg (Female) and 72 kg (Male) which under Duplosian gravity weighs 61.65 kg and 60.8 kg. However because the gravity factors cancel out these averages in Duplosian units are 74.752 dp and 73.728 dp respectively.

There are colloquial terms for other weights but nothing that is universally recognised

3 The ipint is based on an arbitrary container of religious significance that has become the standard. The cubic foot is used but other measures are not derived from it.

4 surface gravity on Duplos is 8.28 m/s^{-2} or 0.8445g

On Isla weight is based on an arbitrary (religious based) standard object - a near spherical ball of iron, of mass 1.823 kg. Thus the average Duplosian woman weighs 40.04 balls. For more domestic convenience island Duplosians use the quarter, which as the name suggests is a quarter of a ball.

Table A3.T4 below summarises the conversion

Table A3.T4: Mass

Geos	Duplos Continent	Duplos Isla	Earth Metric	Earth Imperial
1 swegaweight = ***10^{-6}*** weights			174µg	
1 swegaman = 1 swilipiece = ***10^{-6}*** men = ***10^{-3}*** pieces			25 mg	
1 swiliweight = ***10^{-3}*** weights			0.3 gm	0.01 oz
1 piece = 1 swiliman = ***10^{-3}*** men	0.044 dp	0.095 qr	43.4 gm	1.528 oz
1 weight = ***10*** pieces	0.5325 dp	1.14 qr	520 gm	1.146 lb
1 rock = ***10*** weights	6.39 dp	3.43 balls	6.25 kg	13.752 lb = 0.982 stone
1 **man** = ***10*** rocks	76.68 dp	41.075 balls	75 kg	165 lb = 11 st 11 lb
1 lump = ***10*** men	920 dp	493 balls	900 kg	17.68 cwt
1 twiloman =***1000*** men	132,500 dp	70,978 balls	129,600 kg = 129.6 tonnes	127.28 tons
1.878 weights	1 Dupound	2.143 qr	976.5625 gm	2.152 lb
10.52 pieces	0.4667 dp	1 quarter	455.75 gm	1.0047 lb
3.506 weights	1.8668 dp	1 ball = 4 quarters	1.823 kg	4.019 lb
3.33 swiliweights			1 gm	0.035 oz
0.656 pieces	0.029 dp	0.06245 qr	28.46 gm	1 oz
10.51 pieces	0.4663 dp	0.9992 qr	455.39 gm	1 lb
1.923 weights	1.024 dp	2.194 qr	1 kg	2.196 lb

Angular Measurements

The value of π is universal and the natural radian measure of angle is used in Geosian scientific work just as on Earth. A common approximation to π on Geos is the duodecimal value π = ***3.1848***. Geos has adopted the same degree as Earth (360 = ***260*** degrees in a circle). However further subdivisions are by 72. Thus 72 gemin = 1°, 72 gesec = 1 gemin.

Educated Duplosians are aware of π as the ratio between the circumference of a circle and its diameter and of the area to the square of the radius, they have not developed the concept to include radian measurement of angles. The continentals divide the circle into 60 equal angles, Thus 1 dudeg = 6°.. There is no other universally accepted term for angular measurement other than a right angle which is 15 dudeg. Those wanting more precise measurements use various subdivisions, usually by powers of 2 or powers of 10 or a mixture of the two.

On Isla all divisions are by powers of 2. In the temples they divide a right-angle into 64 idegs. Circular measure is based on practicalities. Those that need to know about such things know that the ratio of the area of a circle to the square of its radius is greater than 3+1/8 but less than 3+3/16. Working with fractions is limited to dividing by 2 (or its powers) and they lack the mathematical notation to deal with fractions.

Temperature on Geos is measured on a scale of ***100*** (144) divisions (div) between the freezing point (***0)*** and boiling point (***100***) of water at sea level on Geos. The Earth Kelvin is 1.44 div Geosian. Geosian science is aware of Absolute Zero.

There are no standardised methods of measuring temperature on Duplos.

Currency

New Norland, the dominant country on Geos, has a three unit duodecimal currency system, for convenience I shall denote them by penny, shilling and pound. As there is no trade between Geos and Earth conversion to Earth currency is pretty meaningless but in terms of wages the pounds are close to 'parity'.

Duplos is even more difficult to compare. Isla's money is issued by the temples, and much of its use is for religious purposes. Much of the economy is based on barter or exchange of obligations. Some money does also circulate, however, and some people are paid wages.

On the continent the various territiories have their own currencies. I shall only give details of that of the Western Federation.

Table A3.T5 gives details of the New Norland currency on Geos.

Table A3.T5: Currency in New Norland

£1 (pound)= 12 ***(10)*** s (shillings) = 144 ***(100)*** d (pence)

	Per Period	Per Work	Per Annum
Minimum Adult Wage	***£497.25***	***£2000***	***£10,000***
(decimal)	£691 2s 5d	£3,456	£20,736
Average Adult Wage	***£6B7.25***	£2,AA0	***£15,000***
(decimal)	£1003 2s 5d	£5,016	£30,096
School Leaver Min	***£324.98***	***£1,400***	***£8,000***
(decimal)	£460 9s 8d	£2,304	£13,824
School Leaver Max	***£449.73***	***£1,A00***	***£A,000***
decimal)	£633 7s 3d	£3,168	£19,008
New Graduate Min	***£56B.25***	***£2,400***	***£12,000***
(decimal)	£803 2s 5d	£4,016	£24,096
New Graduate Max	***£724.98***	***£3,000***	***£16,000***
(decimal)	£1036 9s 8d	£5,184	£31,104

The above shows typical ranges for salaries for school leavers aged ***16*** (18) or new graduates who are likely to be in the age range ***19 - 20*** (21-24). The min and max here are in no way obligatory, but typical. The adult minimum wage applies at age ***20*** (24). Wild Cat Waters and Flying Eagle Forest had starting salaries of ***£4,000*** (£6912) per **work.** But then they did have top qualifications from the leading technical university in the world in a very much sought after subject. My guess is that they were on six figure salaries before too long.

In the Western Federation all money is coin. The main unit of currency is the dollar, the smallest is the farthing. 4 farthings = 1 cent, 4 cents = 1 groat, 10 cents = 1 dime, 25 cents = 1 crown, 2 crowns = 1 dubloon, 2 dubloons = 1 dollar ($1).

Farthings, cents and groats are bronze, dimes, half-crowns, crowns and dubloons are silver. Dollars are silver or gold. $5 and $10 are gold.

There is no minimum wage. Labourers receive between a groat and a dime per horum. An educated woman might expect to earn around $2,000 per annum from her own work on which she will live quite comfortably. At first, Astra paid Arté $500 per selenth of which 50% was deducted for his keep.

On Isla the main unit of currency is the Mark and the smallest the denar, but much of the economy functions without actual money. 4 denari = 1 groat, 8 groats = 1 quarter (mark), 4 quarters = 1 mark. Where wages are paid, a male labourer might get up to 1 groat per ihor. An educated woman in a clerical or professional role might expect a salary of 20 marks per period. Of the coins, anything less than a quarter is bronze, quarters, half marks, mark, and 2 mark coins are silver. There are gold 5, 10, and 20 mark coins.

Appendix 4
Population Statistics and other Details concerning Duplos

Most people on Duplos live in village communities although there are a few towns and a very few larger urban centres on the continent. For convenience I shall define:

City: An urban population centre of more than 10,000 people.

Town: A centre of population beween 1,000 and 10,000 usually associated with a market or specific community activity. On Isla a town is defined as 1024 or more people.

Village: a settled community of population between 50 and 1000, including outlying farms that consider themselves as part of the village community. (On Isla they define a village as a named community of 64 or more people).

Isolated or Unsettled: where dwellings are unsettled (e.g. nomadic) or in isolated communities of fewer than 50 (64 on Isla).

Isla

While there are no countries in the Earth sense the Island, Isla, is dominated by a theocracy within which various 'chiefs' vie for local leadership.

The population of Isla is estimated at around 150,000 (1024 x 128 +1024 x 16). It is hard to be sure. The number of people living in the wild lands is anyone's guess, but there are no real centres of population. A community of 128 would be considered large.

Even in the allied territories of West Isla, New West Isla and the Isla Marches, accurate statistics are hard to come by. In the Theocracy, the temples are supposed to maintain records of births and deaths as well as ownership of males and familial relationships of females, but in practice their performance is patchy. For administrative convenience a community of 1024 or more is designated a town, and one of 64 or more, with a name, a village.

Roughly two thirds of the population live in the Isla Theocracy, having some sort of loose common allegiance. Bordering the Theocracy are the Isla Marches and the territories of West Isla and New West Isla. These territories are in a sense allied to the Theocracy and follow a similar lifestyle. NWI was colonised/conquered by people from West Isla with help and encouragement from the Theocracy and remain politically connected despite poor communications between the two territories. From the point of view of the Theocracy, NWI and the Marches serve to contain the Western Wild Lands, while the Marches also play an important role in limiting communication between the West and North Wild Lands. The Wild Lands are sparsely populated with no towns and few communities justifying the distinction of village. They are seen as a good source of slaves by the Theocracy and its allies.

Estimates of the population distribution on Isla are:

Territory	Population	Number of Towns	Population inTowns	Population in Villages	Isolated Population
Isla Theocracy	101376	15	32768	65536	3072
West Isla	10240	2	2816	6720	704
New West Isla	6528	2	2432	3584	512
Isla Marches	3616	1	1280	2048	288
Wild Lands (N)	20480	0	0	4096	16384
Wild Lands (W)	10240	0	0	2048	8192

The largest town on Isla is Rivermouth, which is mainly on the North bank of the Great River, shortly before it opens in to its estuary. Some linked communities lie on the South bank close to ferries or the bridge.

Approximate populations of the towns in the Theocracy mentioned in Arthay's story are[5]:

Rivermouth: 6,144; Midborough: 4,624; Cattlebridge: 3,456; Chalcliff-on-Strait: 3,072; Honeyfield: 2,176; Oakbridge: 1,408; Barton-within-Lesser: 1,280.

Continent

The continent is divided into 3 powers although the political ties within these powers are very loose and certainly do not prevent internal strife. One power with which we are mainly concerned, The Western Federation is self-contained within reasonably well-defined borders with the other two, following wars that ended around a century previously with agreed border treaties. The other two claim territories extending well beyond their respective administrative reach and generally sparsely populated by people wholly ignorant of the powers that are claiming sovereignty over them[6]. Estimated Statiistics given in the table below ignore these largely uncharted regions. The estimates given for the WF are fairly good but those for the Great Southern Empire and especially the Shadowlands Union are really just best guesses.

Territory	Population	Number of cities	Population in Cities	Population in Towns	Population in Villages	Isolated/ Unsettled Population
Western Federation	1,200,000	8	245,000	250,000	700,000	5,000
Great Southern Empire	1,500,000	5	130,000	320,000	750,000	300,000
Shadowlands Union	2,500,000	6	150,000	530,000	1,200,000	620,000

The approximate population of the cities mentioned in Arté's story are as follows:

Louth: 58,000; Routh: 15,000; Goddess City: 70,000; Marouth: 32,000; Marcenza: 16,000;

Kongholm: 22,000; Dambach: 25,000; Soubanc: 12,500; Lanamont: 10,500.

Below is a distance/ journey time chart for the above eight cities (counting Marouth/Marcenza as one) together with the bridgeheads at Deltaport and Vinebridge, the villages of Islaview, Dunos and South Beach, Observatory Island and Arté's farm at Thayview. Distances are in walks, multiply-

5 Midborough isn't actually mentioned, but it is the second largest town in the Theocracy.

6 Between them the 3 powers claim all territory on Duplos, even though they have no idea of its extent. The Western Federation claims sovereignty over Isla, even though that is an absurd claim.

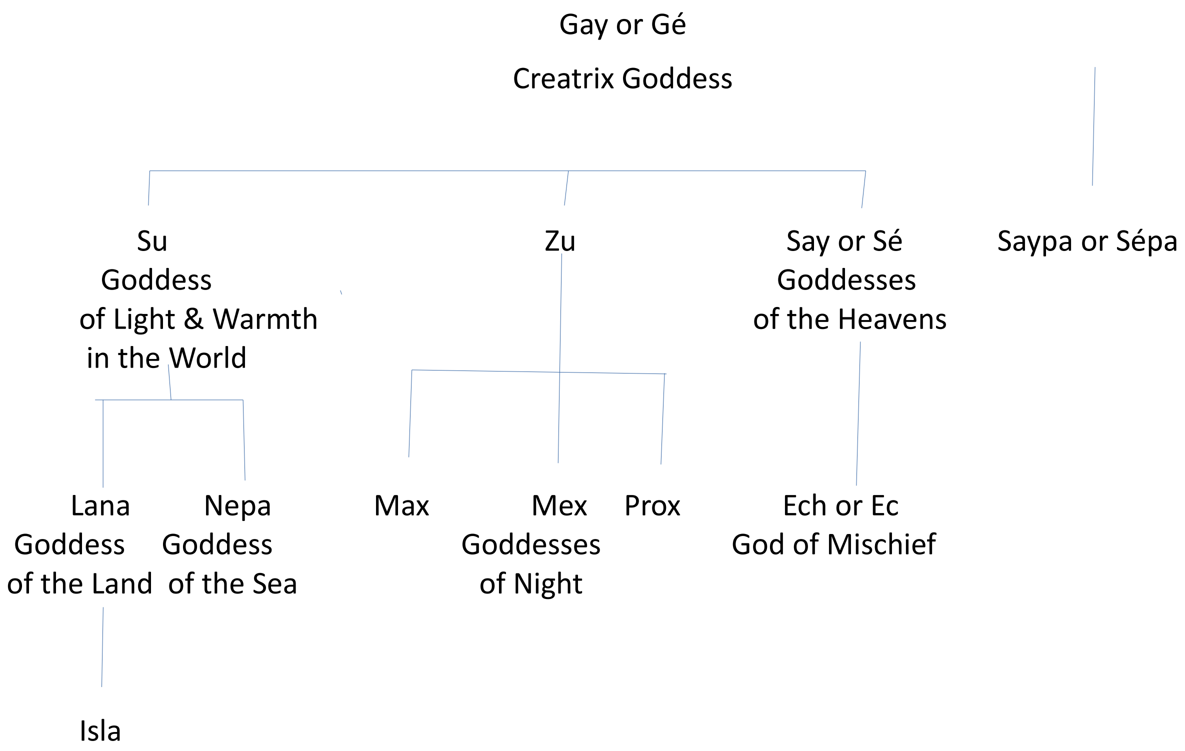
ing by 2.5 converts them to kilometres. Times in hora are Rastu's estimates for the journeys in a two seater self propelled cart with just one person pedalling and one passenger. Distances in walks are in the top right half of the chart, times in hora are in the bottom left in *italics*[7].

Distance & Journey Time Chart

	Louth	Dambach	Deltaport	Dunos	Goddess C.	Islaview	Kongholm	Lanamont	Marouth	Obs. Isl.	Routh	Soubanc	S.Beach	Thayview	Vinebridge
Louth		**66**	**55**	**36**	**69**	**92**	**180**	**205**	**152**	**240**	**35**	**120**	**166**	**74**	**117**
Dambach	**4.2**		17	62	**74**	142	142	238	218	202	44	186	232	118	183
Deltaport	**3.4**	1.1		61	**57**	141	125	221	207	185	41	175	221	119	172
Dunos	**2.4**	4.2	4.4		**98**	80	186	241	188	246	20	151	202	58	153
Goddess C	**4.3**	**4.7**	**3.6**	**6.6**		**161**	**114**	**164**	**214**	**174**	**80**	**230**	**200**	**143**	**180**
Islaview	**6.6**	10.1	10.1	5.7	**11.5**		266	225	175	326	100	121	189	22	130
Kongholm	**11.3**	8.9	7.8	11.8	**7.2**	17.9		254	306	60	166	345	292	254	297
Lanamont	**14.6**	17.0	15.8	17.2	**11.7**	16.1	18.1		50	366	226	140	36	221	103
Marouth	**10.0**	14.5	13.8	12.6	**15.0**	12.5	21.8	3.3		379	174	90	14	183	43
Obs. Isl.	**15.3**	12.9	11.8	15.8	**11.2**	21.9	4.0	22.1	25.8		226	405	352	314	357
Routh	**2.2**	2.8	2.6	1.3	**5.0**	7.1	10.4	16.2	11.6	14.4		155	188	78	152
Soubanc	**8.5**	13.3	12.5	10.8	**16.4**	8.6	24.6	9.8	6.4	28.6	11.1		104	135	54
S. Beach	**10.9**	15.4	14.7	13.5	**14.1**	13.4	20.9	2.4	0.9	24.9	12.5	7.4		197	72
Thayview	**5.0**	8.4	8.5	4.1	**10.2**	1.5	18.2	15.8	13.2	22.6	5.5	9.6	13.4		130
Vinebridge	**7.7**	12.2	11.5	10.9	**12.0**	9.3	21.0	7.3	2.9	25.0	10.1	3.7	3.8	9.3	

7 The distances and timings were provided by Rastu and Zorro, based on a mixture of calculation and experience

Appendix 5
The Duplosian Pantheon and Associated Mythology



Gay (or Gé as she is known on the continent) is the creatrix of the universe. She is too great to be concerned with the details of life on Duplos - but her presence forever looms over all things. She delegated the oversight of her creation to her three daughters, Su, Zu, and Say assisted by her niece Saypa (Su, Zu, Sé, Sépa on the continent).

Su's special responsibility is for the world to which she brings light and warmth but her sisters are not without influence and being as contrary as human sisters bring about night, darkness and cold. Su has two daughters Lana and Nepa and their respective domains are the land and sea. Lana was ever a force for good but Nepa was jealous of her and not being content with her own estates ever seeks to steal her sister's domain and destroy its life. Lana has a daughter, Isla who asked her mother if she might have a part of the land for her own. Lana being a wise mother agreed to give her a peninsula with a fine temperate climate, goodly uniform rainfall and fertile ground where her people might live comfortably and serve her.

This made Nepa especially angry and she made terrble war on her sister and niece. In this war she separated Isla from her mother. The people of Isla saw that they were now alone in the world and resolved to serve their goddess with renewed zeal. However, from time to time, they must make sacrifices to the sea to appease Nepa.

Zu and Say and Saypa made their presence felt more at night and the only male in the pantheon, Ech (Ec on the continent) would cause all sorts of mischief, for he was the wayward and erratic son of Say. (Zu and her daughters (Max, Mex, Prox) play little part in Isla theology although they were more prominent in ancient continental mythology - all dating from before the destruction of the landbridge to Isla).

This religious system that so dominates Isla is no longer followed seriously on the continent but does form part of their mythology and some rituals survive in local cultures.

Appendix 6
Arté's Paper on Islander Arithmetic

Some observations concerning Arithmetic on Isla

by Arté

(University of Routh, Departments of Natural Philosophy and of Ancient History & Theology)[8]

INTRODUCTORY REMARKS

The decimal place value system for representing numbers, now common throughout the continent, has not been discovered on Isla. Numbers are represented by a much more primitive system similar in many ways to archaic number systems that were still in use on the continent before the current system was introduced via the Great Southern Empire about a hundred and fifty years ago.

There are actually three related systems of arithmetic in use on Isla, to my knowledge. The first, and simplest, is in common use throughout the Isla Theocracy and its allied territoties of West Isla, New West Isla and The Isla Marches. This system is taught to all female and a high proportion of male children. The second system uses the same number representation, but improves on the method of multiplication. Knowledge of this is seen as the province of the religious and civil ruling classes and is kept within these circles. I was aware of the existence of these methods while resident on Isla, and had indeed rediscovered them myself some years ago. For confirmation of the details, however, I am grateful to Esau of Islaview Market who in turn received the information from Créezy of Louth, as she had herself been privy to these secrets.

The third method uses a different, but I believe related, number representation and allows for superior multiplication algorithms. The details are closely guarded by a very small elite group of powerful women. This group certainly included Thay, my former mistress, her sister Shay and mother Chay. Créezy is almost certain that her mother Cray (a cousin of Chay's) was also a member of this group but at the time she came to the Western Federation (nearly twelve years ago) her elder sister Craya would not have been. Esau is sure that his former mistress, Shawna, was privy to some improved calculation methods but would have been most unlikely to have been one of the elite. His own mother, Frayna, would almost certainly have had no arithmetic knowledge above normal basics taught in school.

I was never exposed to any details of the third system, but, I had seen some of Thay's calculations in the periods while I was working on the build of her new house, and before she grew suspicious of my interest in arithmetical algorithms. They made little sense to me at the time, but I have a good memory, and with the benefit of my studies here on the continent, I believe I have been able to reconstruct the essential elements of the system. I am certain that none of the methods in use on the island uses a place number system.

BASIC NUMBER REPRESENTATION

There is a symbol for one, unsurprisingly, it is I, other symbols represent powers of two. All these symbols are made up from straight lines, with lower powers of 2 being represented by fewer lines in the symbol, otherwise there doesn't seem to be any particular logic behind the choice of symbol. Numbers are represented by strings of these symbols. The numerical value of each string is just the sum of the values of the symbols in the string.

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8 The Author acknowledges the sponsorship of the Islaview Market Governing Council, for the studies of which this report forms a part.

The symbols in common use today are as follows:

I, L, T, X, V, A, H, K, N, Y, M with respective numerical values 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

Thus the numbers 1 - 20 would be expressed as

I, L, LI, T, TI, TL, TLI, X, XI, XL, XLI, XT, XTI, XTL, XTLI, V, VI, VL, VLI, VT

It is easy to see how numbers build up. However, while the above is a canonical mode of numerical expression, it is not seen as essential. The symbols in a string can be written in any order without affecting the value, and the same symbol can be and often is used more than once in a string. Thus writing XX or TXT for V is perfectly acceptable. Indeed II and III are often used for two and three respectively as is XX for sixteen.

The symbols for 2, 16, 128 and 512 (L, V, K and Y) were only introduced relatively recently, I think about 50 years before I was born, judging by old records I had seen while working for Thay.

It is of historical interest that the symbols I, T and X for 1, 4 and 8 respectively are the same as used in the earliest decimal system used in this part of the Western Federation for the same digits before the standardisation on the GSE symbols in use today. Also I have been assured that ancient clay tablets found recently in a cave in the South-Western part on the Federation indicate that A and H were also used as number symbols and it it reasonable to conclude from the context that they represent the numbers 32 and 64 just as on Isla.

This all indicates a common origin for the number system on Isla and that in use in this part of the continent, suggesting that this method of representing numbers predates the separation.

ARITHMETIC OPERATIONS FOR ORDINARY ISLANDERS

Addition of two numbers of this form is easy, just writing down the two numbers consecutively is a perfectly legitimate representation of the sum. More usefully, if this representation is then sorted with highest values to the left, and then starting at the rightmost end replace every pair of identical symbols with the symbol representing the next higher power of 2 until all symbols are present at most once and you have the sum in canonical form.

Subtraction is slightly more tricky but really quite straight forward. The concept of an algorithm is alien to the common Islander, but in effect the steps below is what they do.

Step 1 start by writing down the *minuend* and *subtrahend* side by side.

Step 2 underneath write down the the same two numbers but with any symbols common to then both deleted.

Step 3 If the are no symbols left on the *subtrahend* side stop, the number on the minuend side is the *difference*.

Step 4 look for the lowest value symbol on the *minuend* side that represents a greater power of 2 than the greatest value symbol left on the *subtrahend* side (if there isn't one then the *subtrahend* is greater than the *minuend* and the subtraction is impossible, Islanders don't do negative numbers, (or zero for that matter). Replace this number in the *minuend* by two copies of the next lower power of 2. Return to Step 2.

Multiplication is much more difficult, and any islander will give up unless the numbers are small. To do multiplication we start by showing how to double. Doubling a number is easy. Start with the number and then replace every symbol by the symbol representing the next higher power of two.

If the symbol M is in the number it means the doubled number is greater than Islanders are willing to cope with. In theory you could just replace M with MM and never need to give up.

To multiply, we start with three columns. Write down the two numbers at the top of the first two columns, it doesn't matter which order but usually it is easier to put either the smaller or the shorter number on the right, it's a matter of experience and personal preference. Draw a line under these,

Below the line, in the second column write down the number from above the line in the first column. and in the third column write a I. Then in both the second and third columns, in the next line write down the double of the number in the previous line. When the number written in the third column reaches the highest value symbol in the number above the line of column 2, stop.

Now go down through the symbols of the number above the line in column 2. For each symbol, find it in column 3 and copy the corresponding number from column 2 to column 1.

When you've gone through all the symbols, add up all the column 1 numbers below the line and that gives you the answer to the multiplication sum.

The complexity of this depends on two things, how many doublings you have to do and how many numbers you have to add up at the end. The first of these depends on the value of the largest symbol at the head of column 2, and the second is the number of symbols in that number.

Example (7 x 5), TLI times TI, (37 x 23) VTLI times ATI (23 x 37)

Columns 1	2	3	1	2	3	1	2	3
TLI	TI		ATI	VTLI		VTLI	ATI	
TLI	TLI	I	ATI	ATI	I	VTLI	VTLI	I
	XTL	L	HXL	HXL	L		AXTL	L
VXT	VXT	T	KVT	KVT	T	HVXT	HVXT	T
				NAX	X		KAVX	X
			YHV	YHV	V		NHAV	V
						YKHA	YKHA	A

TLI+VXT = VXTTLI = ALI (= 35)

Sum = YHVKVTHXLATI
= YKHHAVVXTTLI
=YNHVLI
= 512+256+64+16+2+1=851

Sum = YKHAHVXTVTLI
= YKHHAVVXTTLI
(also =851)

An ordinary Islander would probably give up on multiplying 23 by 37, but could manage 5 by 7.

Contrast this with the ease of multiplying in our decimal system.

Division: Ordinary islanders simply don't do division, halving and quartering are done but that's it. When halving people just about get the rule about remainder 1 for halving an odd number, but they get ever so muddled about the remainder when quartering, which, in my view is down to the way the rules are taught them.

SPEAKING ABOUT NUMBERS

Until the reforms that introduced the missing powers of two, Islish only had words for the numbers up to eight and the powers of two for which they had symbols. They also had the quantifiers: double, quadruple, half and quarter. For the missing powers of two other than two itself for which they had a word, they made use of the double, for example double eight for sixteen. The others would get used under different contexts. Whether the ordinary uneducated person knew that double eight was the same number as half thirty-two was a moot point, which I think was part of the motive for the change, the others being tidiness and removing the need to have repeated symbols when writing a number.

As well as introducing the missing power symbols, and names to go with them, the reforms extended the range of small numbers for which the language had direct expressions to describe them up to 31. Numbers above 32 other than powers of 2, have to be described by a sum like 16 + 32 for 48.

For example when I first arrived here, Esau had to use this convention when talking to me until I had learnt enough to grasp the decimal system.

LARGE NUMBERS

Although in theory any number of any size can be represented if you are willing to use a long enough string of Ms, that is obviously impractical. Thus for ordinary Islanders 1024 is treated as the biggest number. Generally there is no need for them to be concerned about it. More than 1024, or 1024 and then some are the expressions in common use for zillions.

There used not to be many communities that qualified as a town on Isla - more than 1024 in population. Village populations could be counted. Towns couldn't. The literal meaning of the islander words for town could best be translated as 'zillions of people in one place'. As more villages grew over this limit, the authorities found the need to do better. They decided that for the general public, it would be sufficient to use double and quadruple and the recommendation by the reformers to extend with words and symbols up to 8192 was rejected. Although the elite group took on the language, and the symbols did get used in some temple records. For example, population statistics in Rivermouth whose current population is around 6,000. But in public it is still described as quadruple 1024 + double 1024.

SECRET IMPROVEMENTS

As you can tell, multiplication is tedious. To improve this, multiplication tables have been drawn up that make things easier. The theory is to represent either one or both of the multiplicands as sums of numbers for which they have tables, minimising the number of terms in each sum. Together with doubling, this has produced some practical improvements for individual cases. For example combining 3 and 5 times tables with a single doubling gives 6 and 10 times tables[9]. As a minimum both 3 and 5 times tables are in common use, and there are probably others. They also have division tables for 3 and 5 as well as being prepared to implement repeated halving beyond the normal quartering.

That is more or less what I described to Thay when she blew up and stopped me in my tracks. What I didn't telll her was I had a rather more extensive set of tables, and was working on some ideas on what I now know as fractions, that is basically about approximating a fraction by a sum of powers of 1/2. I don't think that even the elite Isla methods have anything to do with that.

For the 23 by 37 example, I would do the equivalent of decomposing it as follows:

23 x 37 = (16+7)(32+5) =512 + 16x5 + 32x7 + 5x7, or perhaps (4x5 + 3)(32+5) =128x5+32x3+4x5x5+3x5 both of which give a simpler calculation given a 5 times table.

9 When I met Créezy, which was after the original publication of this paper, she was able to confirm the use of 3 and 5 times tables and believed there were others but had never had need to know about them.

ELITE METHODS

The top secret approach to arithmetic is effectively a base 8 system but without place value and with no symbol to represent zero - which is what is needed to enable place-value. It works like this. They have just eight symbols for the numbers 1 to 8. They used their existing symbols for 1, 2, 4, and 8, so they just needed to have new ones for 3, 5, 6 and 7. This yielded the set: I, L, Δ, T, Z, F, Π, X. They then use between none and 4 dots with each symbol to multiply its value by eight. So the symbol X with four dots meant 32768. As with their old system numbers were represented by string a of values so X: X would represent 520. They didn't seem to mind that they had two ways of writing eight, they could use X which they invariably did, or I. , Similarly I: and X. both represent 64, but, that aside, they had a base eight system and were able to produce multiplication tables for each of the single symbol numbers and then come up with algorithms which are analagous to the long multiplication and long division algorithms that are used in the continental base ten system. I don't know if they actually had these algorithms but I'm pretty confident that this is the way they represented numbers and it seems logical to assume that that was the motive.

Appendix 7
Arté's Paper on infinitesimals and the infinite

Some philosphical thoughts concerning infinitesimal quantities and the infinite

by Arté

(Astra's House, South Beach, Near Marouth Western Federation[10])

APPROXIMATION OF FRACTIONS BY REPEATED HALVING

On Isla they do not have the concept of fractions as such but undertake division by repeated halving, and in practice rarely go beyond four iterations (division by sixteen).

Thus for most Islanders 1/3 is somewhere beween 5/16 and 3/8. Where greater accuracy is required, there are 'secret' methods for division by 3 and 5 via specially constructed tables, held within the confines of particular religious orders.

When I was still on Isla, I realised that there was no need to be limited to four levels of halving. A fraction is then approximated by a number of iterations with each iteration being an accumulation of halvings, expressed as a double inequality sandwiching the fraction between two fractions with denominators powers of two. For example for 1/3 with 8 iterations yields:

$$0<\frac{1}{3}<\frac{1}{2}\rightarrow\frac{1}{4}<\frac{1}{3}<\frac{2}{4}\rightarrow\frac{2}{8}<\frac{1}{3}<\frac{3}{8}\rightarrow\frac{5}{16}<\frac{1}{3}<\frac{6}{16}\rightarrow\frac{10}{32}<\frac{1}{3}<\frac{11}{32}\rightarrow\frac{21}{64}<\frac{1}{3}<\frac{22}{64}\rightarrow\frac{42}{128}<\frac{1}{3}<\frac{43}{128}\rightarrow$$
$$\frac{85}{256}<\frac{1}{3}<\frac{86}{126}\rightarrow\frac{170}{512}<\frac{1}{3}<\frac{171}{512}$$

giving the final result that 1/3 is between 85/256 and 171/512. The mid value, in this case 341/1024 becomes the 'best' estimate for 1/3.

Once I learned the place value system of representing numbers I realised this amounts to 'decimal' approximations in base 2. In general $c_r/2^r$ is an approximator to the fraction a/b, where,

$c_r = a_0\, 2^r + a_1 2^{r-1} +...+ a_i 2^{r-i} +...+ a_{r-1}2 + a_r$, with the a_i all being 0 or 1, where

$$\frac{a}{b}-\frac{c_r}{2^r}\leq\frac{1}{2^{r+1}}<\frac{c_r+1}{2^r}-\frac{a}{b}$$

MORE GENERAL APPROXIMATION OF FRACTIONS AND UNFRACTIONS

Here in The Western Federation, there is nothing new about this. Indeed what we do when expressing a fraction as a decimal is exactly the above except with ten replacing two in the denominators and any of the digits 0 to 9 occuring as coefficients of the powers of 1/10. The important point it tells us is that it is possible to establish sequences of fractions that sandwich a given number getting closer and closer to the original number, i.e. **infinitesimally** close.

If all numbers could be written as fractions, this insight would tell us nothing, but we know that the square root of 2 cannot be written as a fraction, but from geometry we know we can construct a line of that length from a unit length, being the hypotenuse of a right-angled triangle with

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10 This paper builds on an earlier one by the author written two years previously when a student at the University of Routh. The author acknowledges support and encouragement from Astra for undertaking this work while in her employ. He is also grateful for many conversations with Rastu and with Professor Fermos of the University of Routh.

the other two sides both being of unit length. Now, the point is that for unfractional numbers like the square root of two, we can approach them infinitesimally closely by sequences of fractions as above. Later I'm going to show that far from being atypical the unfractional numbers in a sense outnumber the fractions, even though we only have a very few examples of actual unfractions.[11]

My contention is that any number that can be represented as a point on a line can be approximated to any degree of accuracy required by means of a sequence of fractions approaching it from above or below (that is with fractions all greater or all less that the number concerned). My intention here is to use this fact to construct any such '**line number'** from fractions alone. This is philosophically important because, while the counting numbers can be defined by counting, zero and the negative numbers by subtraction of counting numbers, and the fractional numbers by division by a counting number, we have no actual construction of the line numbers, other than by geometrical construction, which only covers a few cases.

Suppose $s_1, s_2,...$ and $b_1, b_2,...$ are to sequences of fractional numbers satisfying the three properties

1. The sequence {s} gets bigger and the sequence {b} gets smaller as the sequence goes on, i.e $s_i \leq s_j$ and $b_i \geq b_j$ whenever $i < j$.

2. All the numbers in the b sequence are greater than all the numbers in the s sequence, i.e. $s_i < b_j$ for all values of i and j.

3. The two sequences approach each other arbitrarily closely, i.e. suppose e is a small fraction > 0 (for example I might use a large power of 1/10), then there is a number n for which $b_i - s_j < e$ for all values of i and j greater than n.

Then I say that these sequences of fractions define a unique line number, namely the point on the line which each approaches as n increases.

The sequences obtained by writing a number to an increasing number of decimal places are examples of sequences of fractions that satisfy these three properties. The sequence {s} is the sequence of increasing numbers of places in the decimal representation of a number, the sequence b is the same sequence but with the latest decimal place increased by 1. For example the sequences, {s} = {1, 1.7, 1.73, 1.732, 1.732, 1.73205,...} and, {b} ={ 2, 1.8, 1.74, 1.733, 1,7321, 1.73206,.. } are the first few terms of sequences defining the square root of 3.

We know that in the case of the decimal expansion that if the expansion ends in a (finite) recurring sequence then the line number represented is fractional and vice versa.(For example 1/3 = 0.3 recurring and 1/6 = 0.16 with the 6 recurring and 1/7 = 0.142587 all recurring). If the expansion continues without such a recurring tail the number must be unfractional.

Although in practical applications we tend to use these decimal approximations, any sequences of fractions satisfying the three properties can be used to approximate a number.

In the above I have confined my attention to approximating a number by sandwiching it beween two sequences. It is possible to modify the properties to achieve the same result using just one sequence that can arbitrarily jump about between values below and above the number being approximated. As before let {s} be a sequence of fractions. Two properties are now needed:

11 For example those involving the square root of a whole number that is not a perfect square.

1. Eventually the gaps between successive numbers in the sequence get smaller, i.e. there is a number n such that if i, j > n with i > j then the difference between s_{i+1} and $s_i \leq$ the difference between s_{j+1} and s_j (defining difference between a and b to be whichever of a-b or b-a is non-negative).

2. The differences between successive terms approach zero infinitesimally closely, i.e. given any arbitrary positive fraction, e there is a number n such that $-e < s_{i+1} - s_i < e$, for all values of i > n.

CONTEMPLATING THE INFINITE

There is an ancient paradox involving a race between a horse and donkey. The horse runs twice as fast as the donkey, which is therefore given a start of, say, one unit. After a while the horse reaches the donkey's starting position, but the donkey is still ahead by half a unit. By the time the horse reaches that point the donkey has moved on and is still ahead, this goes on. How does the horse ever catch the donkey? The point is, of course, that both donkey and horse are moving on a continuous line and approximating by small steps is inappropriate. This got me thinking about different approaches to the infinite.

First there is the infinite like the infinite number of counting numbers. This is weird enough. Because we can leave one out and still have an infinite number left. We can even leave half of them out and stay with the same infinite number. For example we can count the even numbers using all the counting numbers, even though the even numbers themselves are just half of all the numbers. We can count them thus:

Even Numbers: 2, 4, 6, 8 ,10,12,.......

Counting them: 1, 2, 3, 4, 5, 6,.........

We can even count all pairs of numbers, to see this consider setting them out in a square array, as illustrated here:

	1	2	3	4	5	......
1	1, 1	1, 2	1, 3	1, 4	1, 5	.....
2	2, 1	2, 2	2, 3	2, 4	2, 5	.....
3	3, 1	3, 2	3, 3	3, 4	3, 5	......
4	4, 1	4 2	4, 3	4, 4	4, 5.	......
5.	5, 1	5, 2	5, 3	5, 4	5, 5	......

Now starting in the top left corner we can count the pairs as:

Counting:	1	2	3	4	5	6	7	8	9	10	11	12
Pairs:	1, 1	1, 2	2, 1	3, 1	2, 2	1, 3	1, 4	2, 3	3, 2	4, 1	5, 1	4, 2 etc.

This way we see that we can count the pairs of numbers using the counting numbers without missing any out or running out of numbers to count with. Since fractions are just pairs of numbers this means we can also count the fractional numbers, even though we can't list them in order of size.

This is the prelude to my next philosophical discovery. You can't count the line numbers, no matter how you try you will always miss some out. I demonstrate this as follows. To make things easy I'll just try to count the line numbers between 0 and 1. Suppose I can do so, then I can list them in some order 1, 2, 3 etc.

I'm now going to construct a line number between 0 and 1 that is not in the list. Before the decimal point the number will be 0.

After the decimal point I proceed as follows:

Take the first decimal place of the first number in the list, it will contain a number between 0 and 9, a_1, say. I am going to chose a different value, for technical reasons I will avoid choosing 0 or 9. To be definite, I will choose $b_1 = a_1 +1$ unless a_1 is 8 or 9 when I will choose $b_1 = 1$. This choice will then be the first value after the decimal point in my constructed number.

Do the same with the second number on the list, this time taking the second decimal place, changing it as above to get b_2.

Continue in the same way, fom the nth decimal place of the nth number in the list choose b_n to be different.

Then I will end up with a number $0.b_1b_2b_3...b_n....$ which is a number between 0 and 1. It is also not in my original list because it differs from each number in the list, at least in one decimal place.

This is impossible if I had indeed counted all the line numbers between 0 and 1. That means that the infinity of the line numbers is bigger than the infinity of the counting numbers. But the infinity of the fractional numbers is the same as the infinity of the counting numbers. This means there is a bigger infinity of unfractions than of fractions.

This seems wierd because a different argument can be used to show that between any two unfractions there is at least one fraction (in fact there will be an infinite number of them).

To see this consider two different numbers (fractions or unfractions) $a < b$.

Let $b-a = c > 0$. Now take fractional approximations $a_f \geq a$ and $b_f \leq b$ respectively that are closer than $c/3$. Then $(b_f - a_f)/2$ is a fractional number between a and b. Indeed the line segment between these fractional numbers is a fractional number greater than $c/3$ which is non-zero and will thus contain infinitely many other fractional numbers. I said it was weird.

What this means is that when dealing with infinity, care needs to be taken on whether we are dealing with continuous processes like, for example, the asymptotic behaviour of a hyberbola, or discrete processes like counting.