Some observations concerning Arithmetic on Isla

by
Arté

(University of Routh, Departments of Natural Philosophy and of Ancient History & Theology)[1]

INTRODUCTORY REMARKS

The decimal place value system for representing numbers, now common throughout the continent, has not been discovered on Isla. Numbers are represented by a much more primitive system similar in many ways to archaic number systems that were still in use on the continent before the current system was introduced via the Great Southern Empire about 150 years ago.

There are actually 3 related systems of arithmetic in use on Isla, to my knowledge. The first, and simplest, is in common use throughout the Isla Theocracy and its allied territoties of West Isla, New West Isla and The Isla Marches. This system is taught to all female and a high proportion of male children. The second system is uses the same number representation, but improves on the method of multiplication. Knowledge of this is seen as the province of the Religious and Civil ruling classes and is kept within these circles. I was aware of the existence of these methods while resident on Isla, and had indeed rediscovered them myself some years ago. For confirmation of the details, however I am grateful to Esau of Islaview Market who in turn received the information from Créezy of Louth, as she had herself been privy to these secrets.

The third method uses a different, but I believe related, number representation and allows for superior multiplication algorithms. The details are closely guarded by a very small elite group of powerful women. This group certainly included Thay, my former mistress, her sister Shay and mother Chay. Créezy is almost certain that her mother Cray (a cousin of Chay's) was also a member of this group but at the time she came to the Western Federation (nearly 12 years ago) her elder sister Craya would not have been. Esau is sure that his former mistress, Shawna, was privy to some improved calculation methods but would have been most unlikely to have been one of the elite. His own mother, Frayna, would almost certainly had no arithmetic knowledge above normal basics taught in school.

I was never exposed to any details of the third system, but, I had seen some of Thay's calculations in the periods while I was working on the build of her new house, and before she grew suspicious of my interest in arithmetical algorithms. They made little sense to me at the time, but I have a good memory, and with the benefit of my studies here on the continent, I believe I have been able to reconstruct the essential elements of the system. I am certain that none of the methods in use on the island use a place number system.

BASIC NUMBER REPRESENTATION

There is a symbol for one, unsurprisingly, it is I, other symbols represent powers of two. All these symbols are made up from straight lines, with lower powers of 2 being represented by fewer lines in the symbol, otherwise the doesn't seem to be any particular logic behind the choice of symbol. Numbers are represented by strings of these symbols.

---

1 The Author acknowledges the sponsorship of the Islaview Market Governing Council, for the studies of which this report forms a part.

The numerical value of each string is just the sum of the values of the symbols in the string.

The symbols in common use today are as follows:

I, L, T, X, V, A, H, K, N, Y, M with respective numerical values 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

Thus the numbers 1 - 20 would be expressed as
I, L, LI, T, TI, TL, TLI, X, XI, XL, XLI, XT, XTI, XTL, XTLI, V, VI, VL, VLI, VT

It is easy to see how numbers build up. However, while the above is a canonical mode of numerical expression, it is not seen as essential. The symbols in a string can be written in any order without affecting the value, and the same symbol can be and often is used more than once in a string. Thus writing XX or TXT for V is perfectly acceptable. Indeed II and III are often used for 2 and 3 respectively as is XX for 16.

The symbols for 2, 16, 128 and 512 (L, V, K and Y) were only introduced relatively recently, I think about 50 years before I was born, judging by old records I had seen while working for Thay.

It is of historical interest that the symbols I, T and X for 1, 4 and 8 respectively are the same as used in the earliest decimal system used in this part of the Westen Federation for the same digits before the standardisation on the GSE symbols in use today. Also I habe been assured that ancient clay tablets found recently in a cave in the South-Western part on the Federation indicate that A and H were also used as number symbols and it it reasonable to conclude from the context that they represent the numbers 32 and 64 just as on Isla.

This all indicates a common origin for the number system on Isla and that in use in this part of the continent, suggesting that this method of representing numbers predates the separation.

ARITHMETIC OPERATIONS FOR ORDINARY ISLANDERS

Addition of two numbers of this form is easy, Just writing down the two numbers consecutively is a perfectly legitimate representation of the sum. More usefully, if this representation is then sorted with highest values to the left, and then starting at the rightmost end replace every pair of identical symboles with the symbol representing the nest higher power of 2 until all symbols are present at most once and you have the sum in canonical form.

Subtraction is slightly more tricky but really quite straight forward. The concept of an algorithm is alien to the common Islander, but in effect the steps below is what they do.

Step 1; start by writing down the *minuend* and *subtrahend* side by side.
Step2 underneath write down the the same two numbers but with any symbols common to then both deleted.
Step 3 If the are no sybols left on the *subtrahend* side stop, and the *difference* is your answer.
Step 4b look for the lowest value symbol on the *minuend* side that represents a greater power of 2 than the greatest value symbol left on the *subtrahend* side (if there isn't one

then the *subtrahend* is greater than the *minuend* and the subtraction is impossible, Islanders don't do negative numbers, or zero for that matter)
Replace this number in the *minuend* by two copies of the next lower power of 2. Return to Step 2.

Multiplication is much more difficult, and any islander will give up unless the numbers are very small. To do multiplication we start by showing how to double. Doubling an number is easy. Start with the number and then replace every symbol by the symbol representing the next higher power of two. If the symbol M is in the number it means the doubled number is greater than Islanders are willing to cope with. In theory you could just replace M with MM and never need to give up.

To multiply, we start with three columns. Write down the two numbers at tthe top of the first two columns, it doesn't matter which order but usually it is easier to put either the smaller or the shorter number on the right, it's a matter of experience and personal preference. Draw a line under these,

Below the line, in the second column write down the number from above the line in the first column. and in the third column write a I. Then in both the second and third columns, in the next line write down the double of the number in the previous line. When the number written in the third column reaches the highest value symbol in the number above the line of column 2, stop.

Now go down through the symbols of the number above the line in column 2. For each symbol, find it in column 3 and copy the corresponding number from column 2 to column 1.

When you've gone through all the symbols, add up all the column 1 numbers below the line and that gives you the answer to the multiplication sum.

The complexity of this depends on two things, how many doublings do you have to do and how many numbers do you have to add up at the end. The first of these depends on the value of the largest symbol at the head of column 2, and the second is the number of symbols in that number.

Example (7 x 5), TLI times TI,
Columns 1 2 3

1	2	3
TLI	TI	
TLI	TLI	I
	XTL	L
VXT	VXT	T

TLI+VXT = VXTTLI
= ALI (= 35)

(37 x 23) VTLI times ATI

1	2	3
ATI	VTLI	
ATI	ATI	I
HXL	HXL	L
KVT	KVT	T
	NAX	X
YHV	YHV	V

Sum = YHVKVTHXLATI
= YKHHAVVXTTLI
=YNHVLI
= 512+256+64+16+2+1=851

(23 x 37)

1	2	3
VTLI	ATI	
VTLI	VTLI	I
	AXTL	L
HVXT	HVXT	T
	KAVX	X
	NHAV	V
YKHA	YKHA	A

Sum = YKHAHVXTVTLI
= YKHHAVVXTTLI
(also =851)

An ordinary Islander would probably give up on multiplying 23 by 37, but could manage 5 by 7.

Contrast this with the ease of multiplying in our decimal system.

Division: Ordinary islanders simply don't do division, halving and quartering are done but that's it. When halving people just about get the rule about remainder 1 for halving an odd number, but they get ever so muddled about the remainder when quartering, which, my view is down to the way the rules are taught them.

SPEAKING ABOUT NUMBERS

Until the reforms that introduced the missing powers of 2, Islish only had words for the numbers up to 8 and the powers of two for which they had symbols. They also had the quantifiers: double, quadruple, half and quarter. For the missing powers of two other than two itself for which they had a word, they made use of the double, for example double eight for sixteen. The others would get used under different contexts. Whether the odinary uneducated person knew that double eight was the same number as half thirtytwo was a moot point, which I think was part of the motive for the change, the others being tidiness and removing the need to have repeated symbols when writing a number.

As well as introducing the missing power symbols, and names to go with them, the reforms extended the range of small numbers for which the language had direct expressions to describe them up to 31. Numbers above 32 other than powers of 2, have to be described by a sum like 16 + 32 for 48.

For example when I first arrived here, Esau had to use this convention when talking to me until I had learnt enough to grasp the decimal system.

LARGE NUMBERS

Although in theory any number of any size can be represented if you are willing to use a long enough string of Ms, that is obviously impractical. Thus for ordinary Islanders 1024 is treated as the biggest number. Generally there is no need for them to be concerned about it. More than 1024, or 1024 and then some are the expressions in common use for zillions.

There used not to be many communities that qualified as a town on Isla - more than 1024 in population. Village populations could be counted. Towns couldn't. The literal meaning of the islander words for town could best be translated as 'zillions of people in one place'. As more villages grew over this limit, the authorities found the need to do better. They decided that for the general public, it would be sufficient to use double and quadruple and the recommendation by the reformers to extend with words and symbols up to 8192 was rejected. Although the elite group took on the language, and the symbols did get used in some temple records, for example, population statistics in Rivermouth whose current population is around 6,000. But in public it is still described as quadruple 1024 + double 1024.

SECRET IMPROVEMENTS

As you can tell multiplication is tedious. To improve this, multiplication tables have been drawn up that make things easier. The theory is to represent either one or both of the multiplicands as sums of numbers for which they have tables, minimising the number of terms in each sum. Together with doubling, this had produced some practical improvements for individual cases. For example combining 3 and 5 times tables with a single doubling gives 6 and 10 times tables. As a minimum both 3 and 5 times tables are in common use, and Créezy says she believes there are others but had never had need to know about them. They also have division tables for 3 and 5 as well as being prepared to implement repeated halving beyond the normal quartering.

That is more or less what I described to Thay when she blew up and stopped me in my tracks. What I didn't telll her was I had a rather more extensive sets of tables, and was working on some ideas on what I now know as fractions, that is basically about approximating a fraction by a sum of powers of 1/2. I don't think that even the elite Isla methods have anything to do with that.

For the 23 by 37 example, I would do the equivalent of decomposing it as follows: 23 x 37 = (16+7)(32+5) =512 + 16x5 + 32x7 + 5x7, or perhaps (4x5 + 3)(32+5) =128x5+32x3+4x5x5+3x5 both of which give simpler calculation given a 5 times table.

ELITE METHODS

The top secret approach to arithmetic was effectively a base 8 system but without place value and with no symbol to represent zero - which is what is needed to enable place-value. It worked like this. They had just 8 symbols for the numbers 1 to 8. They used their existing symbols for 1, 2, 4, and 8, so they just needed to have new ones for 3, 5, 6 and 7. This yielded the set: I, L, Δ, T, Z, F, Π, X. They then used between none and 4 dots with each symbol to multiply its value by 8. So they symbol X with 4 dots meant 4096. As with their old system numbers were represented by string of values so X: X would represent 520. They didn't seem to mind that they had two ways of writing 8, they could use X which they invariably did, or I. , Similarly I: and X. both represent 64. But that aside they had a base 8 system and were able to produce multiplication tables for each of the single symbol numbers and then come up with algorithms which are analagous to the long multiplication and long division algorithms that are used in the continental base 10 system. I don't know if they actually had these algorithms but I'm pretty confident that this is the way they represented numbers and it seems logical to assume that that was the motive.